A Mathematical Refutation of "The product rule, cross product, disparity"

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Abstract

This paper presents a formal academic critique of the mathematical claims made in "The product rule, cross product, disparity". The original text argues that applying the product rule to the cross product operator is mathematically invalid because it ostensibly neglects the angle between the variables. This critique demonstrates that the author's conclusion relies on conflating the vector cross product with its scalar magnitude. By reviewing standard vector calculus, we show that component-wise differentiation inherently accounts for the changing angle between vectors, proving that the established application of the product rule is correct and does not rely on an implicit assumption of a fixed angle.

1 Introduction

In this critique, we analyze the arguments presented in the paper "The product rule, cross product, disparity" [1]. The document posits that a ubiquitous mathematical operation is fundamentally flawed [1]. The author claims that applying the product (Leibniz) rule to the cross product operator is a "stupid mistake" and asserts that this operation is invalid [1]. This paper will deconstruct the mathematical reasoning behind this assertion, highlight the algebraic misunderstandings at its core, and explain why standard vector calculus perfectly accounts for the geometry in question.

2 Analysis of the Original Argument

The core argument of "The product rule, cross product, disparity" rests on the geometric definition of the cross product [1]. The author outlines the following key points:

- The cross product of two variables, given in the text as $r \times p$, is not merely the product of r and p [1].
- There is a "hidden factor" carried along with the cross product operator: the angle between the variables [1].
- The operation is specifically defined by the author as the product of r and $p \sin(\theta)$ [1].
- The central thesis claims that current mathematical usage of the product rule upon the cross product neglects this angle, acting as an implicit assumption that $\sin(\theta)$ equals 1 [1].

Based on these premises, the author concludes that applying the product rule to the cross product restricts its application nonsensically [1].

3 Mathematical Refutation

The error in the source material stems from conflating the *vector* cross product with its *scalar* magnitude.

The cross product of two time-dependent vectors, $\vec{r}(t)$ and $\vec{p}(t)$, is defined algebraically by its Cartesian components (e.g., utilizing the standard basis vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$), not solely by its geometric magnitude. When applying the derivative with respect to a scalar such as time (t), the product rule operates component-wise:

$$\frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \tag{1}$$

Because the differentiation is distributed across orthogonal basis components, the changing angle $\theta(t)$ between the two vectors is inherently tracked and accounted for. The complete geometry of the vectors—including their changing magnitudes and the shifting angle between them—is fully encapsulated by their constituent components.

The author's defining expression, "r and $p \sin(\theta)$" [1], actually describes the *magnitude* of the cross product vector:

$$|\vec{r} \times \vec{p}| = |\vec{r}||\vec{p}| \sin(\theta) \tag{2}$$

If one were to differentiate this *scalar* quantity with respect to time to find the rate of change of the magnitude, the standard scalar product rule (extended across three variables) must indeed be used, yielding:

$$\frac{d}{dt}(|\vec{r}||\vec{p}|\sin\theta) = \frac{d|\vec{r}|}{dt}|\vec{p}|\sin\theta + |\vec{r}|\frac{d|\vec{p}|}{dt}\sin\theta + |\vec{r}||\vec{p}|\cos\theta\frac{d\theta}{dt} \tag{3}$$

The established vector product rule does not "neglect" the angle as the author claims [1]. Rather, it bypasses the need to isolate the angle explicitly by operating directly on the vectors themselves. By differentiating the vectors mathematically, the change in the angle $\frac{d\theta}{dt}$ is implicitly resolved. Therefore, the assumption that $\sin(\theta) = 1$ is never made in standard vector differentiation.

4 Conclusion

The assertion that the product rule is mathematically invalid when applied to the cross product operator is incorrect. The author of "The product rule, cross product, disparity" fails to distinguish between differentiating a vector operation and differentiating its scalar magnitude equivalent [1]. Standard vector calculus accurately captures all geometric dynamics, including angular change, through linear component-wise differentiation, proving that the traditional application of the product rule is neither a mistake nor an implicit assumption of a fixed angle.

References

[1] Mandibaur, J.H. "The product rule, cross product, disparity." Baur Research CC, 201 Republic Road, Randburg, South Africa. Available at: `https://www.researchgate.net/publication/361388725_The_product_rule_cross_product_disparity`.