

An Analysis of the Claim That Light Possesses Mass: A Rebuttal to Mandlbaur

Dave Voorhis and Google Gemini

Abstract

This paper critically evaluates the mathematical and physical claims presented in “A Mathematical Proof That Light Has Mass” by John H. Mandlbaur (2022). We demonstrate that the author’s argument fails on four independent grounds. First, from the standpoint of formal logic and elementary algebra, the proof conflates a tautological identity ($0 = 0$) with a logical contradiction and violates the non-zero precondition required for algebraic cancellation, mistaking a removable singularity for a physical impossibility. Second, from the standpoint of relativistic mechanics, equating $F = ma$ with Newtonian gravitation commits a category error by applying non-relativistic, massive-particle dynamics to relativistic quanta, where any non-zero rest mass would strictly forbid propagation at the invariant speed c . Third, while classical Newtonian gravitation extended via the Equivalence Principle predicts a solar light-deflection angle of 0.875 arcseconds, Einstein’s General Relativity predicts 1.75 arcseconds along null geodesics—a value decisively confirmed by solar eclipse observations and modern radio interferometry, directly falsifying the Newtonian gravitational equation upon which Mandlbaur’s proof depends. Finally, we review independent electrodynamic and astrophysical measurements that constrain the photon rest mass to $m_\gamma < 1.8 \times 10^{-54}$ kg.

1 Introduction

In the short paper “A Mathematical Proof That Light Has Mass,” Mandlbaur [1] asserts that the established scientific consensus that light has zero rest mass relies on an “unsupported assumption” and a “logical fallacy.” By equating Newton’s Second Law of Motion with Newton’s Law of Universal Gravitation, Mandlbaur attempts a proof by contradiction (*reductio ad absurdum*), arguing that setting the mass of a particle to zero necessitates dividing by zero when solving for gravitational acceleration. Because division by zero is undefined, the author concludes that a massless photon is mathematically impossible.

In this paper, we deconstruct Mandlbaur’s argument step by step. We show that the purported contradiction is an artifact of an invalid algebraic operation performed by the author, and that both theoretical consistency and high-precision experimental physics rule out the Newtonian premise and confirm the masslessness of the photon.

2 Reconstruction of Mandlbaur’s Argument

Mandlbaur’s proof [1] consists of four numbered equations drawn from classical Newtonian mechanics:

$$F = ma \tag{1}$$

$$F = G \frac{m_1 m_2}{r^2} \tag{2}$$

$$ma = G \frac{Mm}{r^2} \tag{3}$$

$$a = G \frac{M}{r^2} \tag{4}$$

where F is the magnitude of the net force, m (replacing m_2) is the inertial and gravitational mass of the test particle, a is its acceleration, G is the Newtonian gravitational constant, M (replacing m_1) is the mass of the attracting body, and r is the center-of-mass separation.

Mandlbaur contends that modern physics uses the cancellation of m between equation (3) and equation (4) to claim that mass is irrelevant to gravitational trajectory, thereby allowing light to be deflected by gravity while having $m = 0$. He argues that if $m = 0$, the transition from equation (3) to equation (4) requires dividing by zero—an “unrealistic possibility”—and concludes that “it is impossible for light to have zero mass” [1].

3 Mathematical and Logical Refutation

Even before considering modern physics, Mandlbaur’s deduction fails within elementary algebra and formal logic for three reasons.

3.1 Failure of the *Reductio ad Absurdum*

A valid proof by contradiction requires assuming a proposition P (here, $m = 0$) and applying valid rules of inference to derive a logical contradiction of the form $Q \wedge \neg Q$ (such as $1 = 0$).

When we substitute the hypothesis $m = 0$ directly into Mandlbaur’s starting equality, Eq. (3), we obtain:

$$(0)a = G \frac{M(0)}{r^2} \implies 0 = 0 \quad (5)$$

The statement $0 = 0$ is a mathematical tautology, not a contradiction. It is identically true for all real values of $a \in \mathbb{R}$. Because no contradiction arises from substituting $m = 0$ into Eq. (3), the *reductio ad absurdum* fails immediately.

3.2 Violation of the Algebraic Cancellation Precondition

In real algebra, the multiplicative cancellation law states that for real numbers $m, x, y \in \mathbb{R}$:

$$mx = my \implies x = y \quad \text{if and only if} \quad m \neq 0 \quad (6)$$

Dividing both sides of Eq. (3) by m to obtain Eq. (4) is an optional algebraic manipulation chosen by the analyst, and it carries the explicit domain restriction $m \neq 0$. Assuming $m = 0$ and subsequently dividing by m violates the precondition of the division operator, yielding the indeterminate form $a = \frac{0}{0}$.

Blaming the physical hypothesis ($m = 0$) for an undefined result caused by applying an algebraic operation outside its valid domain is a formal mathematical fallacy. The indeterminacy of $\frac{0}{0}$ merely signifies that the unreduced force equation $0 \cdot a = 0$ carries no information about the kinematic acceleration a of a massless entity; it does not prove that m cannot be zero.

3.3 Removable Singularity and Continuous Extension

If one wishes to study the behavior of classical gravitational acceleration as the test mass vanishes, elementary calculus provides the rigorous tool: the mathematical limit. Treating the gravitational acceleration $a(m)$ as a function on the domain $m \in (0, \infty)$:

$$a(m) = \frac{F_{\text{grav}}(m)}{m} = \frac{1}{m} \left(G \frac{Mm}{r^2} \right) \quad (7)$$

the point $m = 0$ is a classic *removable singularity*. Taking the one-sided limit as the mass approaches zero from above yields a finite, exact, and unambiguous value:

$$\lim_{m \rightarrow 0^+} a(m) = \lim_{m \rightarrow 0^+} \left(G \frac{M}{r^2} \right) = G \frac{M}{r^2} \quad (8)$$

Because the limit exists and is independent of m , $a(m)$ admits a unique continuous extension to $m = 0$. This exact limit is the mathematical basis of the Weak Equivalence Principle in classical mechanics: a test particle's gravitational trajectory is independent of its mass even in the massless limit.

4 Physical and Empirical Refutation

Beyond its algebraic flaws, Mandlbaur's paper commits a fundamental category error by treating classical Newtonian mechanics as universally valid for light, ignoring both relativistic dynamics and over a century of experimental falsification.

4.1 Inapplicability of $F = ma$ and Relativistic Dispersion

Newton's Second Law in the form $F = ma$ is a non-relativistic approximation valid only for massive bodies moving at speeds much smaller than the speed of light ($v \ll c$). Even in classical mechanics, the more fundamental definition of force is the time rate of change of momentum, $\mathbf{F} = \frac{d\mathbf{p}}{dt}$.

In Special Relativity, a particle of invariant rest mass m_0 moving at velocity $\mathbf{v}$ has relativistic energy $E = \gamma m_0 c^2$ and momentum $\mathbf{p} = \gamma m_0 \mathbf{v}$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor. If light possessed any non-zero rest mass ($m_0 > 0$), accelerating a photon to the speed of light ($v = c$) would cause $\gamma \rightarrow \infty$, requiring infinite energy. Consequently, a particle can travel at the invariant speed c in a vacuum if and only if its rest mass is strictly zero ($m_0 = 0$).

The general relativistic dispersion relation connecting energy E , momentum $p = |\mathbf{p}|$, and invariant rest mass m_0 is:

$$E^2 = (pc)^2 + (m_0 c^2)^2 \quad (9)$$

For a photon ($m_0 = 0$), this reduces to $E = pc$. Combined with the Planck–Einstein relation $E = h\nu$, a photon carries well-defined momentum $p = \frac{h\nu}{c}$ and energy without possessing rest mass.

4.2 Gravitational Light Deflection: Newtonian Gravity vs. General Relativity

It is particularly instructive to examine what happens if one actually applies Mandlbaur's equation (4), $a = G\frac{M}{r^2}$, to a ray of light grazing a massive body of mass M at impact parameter b with speed c . This exact Newtonian calculation was performed by Johann Georg von Soldner in 1801 [2]. Integrating the transverse Newtonian acceleration along the trajectory yields a total deflection angle of:

$$\theta_{\text{Newton}} = \frac{2GM}{c^2 b} \quad (10)$$

For starlight grazing the limb of the Sun ($M = M_\odot$, $b = R_\odot$), Newtonian gravity predicts a deflection of $\theta_{\text{Newton}} \approx 0.875$ arcseconds.

In contrast, modern physics does not derive the gravitational bending of light by cancelling m in $ma = G\frac{Mm}{r^2}$. Instead, gravitational interaction is governed by Einstein's General Relativity [3], in which gravity is not a Newtonian force acting on mass, but the curvature of spacetime sourced by the stress-energy tensor $T_{\mu\nu}$. Massless photons possess non-zero energy and momentum ($T_{\mu\nu} \neq 0$) and follow null geodesics ($ds^2 = 0$) governed by the geodesic equation:

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0 \quad (11)$$

where λ is an affine parameter and $\Gamma_{\alpha\beta}^\mu$ are the Christoffel symbols of the spacetime metric. Notice that test-particle mass never enters the geodesic equation at all. Because General Relativity accounts for both temporal metric curvature (which reproduces the Newtonian value)

and spatial metric curvature in the Schwarzschild geometry, it predicts a deflection angle for massless particles that is exactly twice the Newtonian prediction:

$$\theta_{\text{GR}} = \frac{4GM}{c^2 b} \approx 1.75 \text{ arcseconds} \quad (12)$$

Observations of starlight deflection during the total solar eclipse of May 29, 1919, led by Dyson, Eddington, and Davidson [4], decisively confirmed Einstein’s prediction of 1.75 arcseconds and falsified the Newtonian prediction of 0.875 arcseconds. Modern Very Long Baseline Interferometry (VLBI) measurements of radio waves passing near the Sun have since confirmed the General Relativistic deflection formula to within 0.01% [5]. Thus, the classical Newtonian gravitational framework ($ma = G\frac{Mm}{r^2}$) upon which Mandlbaur relies is empirically falsified for light (see table 1).

Table 1: Comparison of the Newtonian corpuscular assumption used in Mandlbaur [1] versus established relativistic physics.

Physical property	Newtonian model ($m > 0$)	Relativistic physics ($m_0 = 0$)
Governing law of motion	$F = ma = G\frac{Mm}{r^2}$	Null geodesic: $\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0$
Energy–momentum relation	$E_k = \frac{1}{2}mv^2, p = mv$	$E = pc = h\nu$
Behavior at $v = c$	Unbounded relativistic energy ($\gamma \rightarrow \infty$)	Exact invariant vacuum propagation speed
Solar limb light deflection	$\theta_N = \frac{2GM_\odot}{c^2 R_\odot} \approx 0.875''$ (Falsified)	$\theta_{\text{GR}} = \frac{4GM_\odot}{c^2 R_\odot} \approx 1.75''$ (Confirmed)

4.3 Electrodynamic and Astrophysical Bounds on Photon Mass

Finally, assigning a non-zero rest mass to light has severe, testable consequences in electrodynamics. In standard quantum electrodynamics and classical electromagnetism, $U(1)$ gauge invariance requires the photon to be strictly massless ($m_\gamma = 0$). If the photon possessed a non-zero mass $m_\gamma > 0$, Maxwell’s equations would be replaced by Proca electrodynamics, resulting in three observable phenomena:

1. **Vacuum dispersion:** The group velocity of light in a vacuum would depend on its frequency ($v_g(\nu) = c\sqrt{1 - (m_\gamma c^2/h\nu)^2}$), meaning high-frequency gamma rays or blue light from distant astrophysical events would arrive significantly earlier than low-frequency radio waves.
2. **Yukawa decay of static fields:** Electrostatic and magnetostatic potentials would decay exponentially over a characteristic Compton wavelength $\lambda_c = \frac{\hbar}{m_\gamma c}$, altering Coulomb’s inverse-square law and the large-scale structure of planetary and solar magnetic fields.
3. **Longitudinal polarization:** Massive vector bosons possess three polarization states rather than the two transverse polarizations observed in electromagnetic radiation.

High-precision tests of Coulomb’s law, satellite measurements of solar wind magnetohydrodynamics, and timing observations of Fast Radio Bursts have detected zero evidence of photon mass, placing an upper bound compiled by the Particle Data Group at $m_\gamma < 10^{-18} \text{ eV}/c^2 \approx 1.8 \times 10^{-54} \text{ kg}$ [5].

5 Conclusion

Mandlbaur’s claim that light must possess mass is incorrect on every mathematical and physical level. Mathematically, substituting $m = 0$ into $ma = G \frac{Mm}{r^2}$ produces the tautology $0 = 0$, not a contradiction; the undefined division by zero cited by the author is a self-inflicted violation of algebraic domain preconditions, whereas the calculus limit $\lim_{m \rightarrow 0^+} a(m) = G \frac{M}{r^2}$ is completely well-behaved. Physically, light is not governed by $F = ma$ or Newtonian gravitation: modern physics derives gravitational lensing from the null geodesics of General Relativity without ever invoking or cancelling a photon mass term. Empirical observations of solar light deflection and stringent tests of electrodynamics decisively refute Mandlbaur’s premise and confirm that the photon is massless.

References

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